W 866

NATURE'S SECRET CODE

Fibonacci Sequences, Spirals and the Golden Ratio

Jennifer DeBruyn, Associate Professor, Department of Biosystems Engineering and Soil Science
University of Tennessee

Nature's Secret Code

Fibonacci Sequences, Spirals and the Golden Ratio

Skill Level
Intermediate-Advanced, 7th - 12th Grade

Learner Outcomes
The learner will be able to:
• Understand how to derive a Fibonacci sequence
• Identify Fibonacci numbers in nature
• Know what the golden ratio is (phi) and identify examples in nature

Educational Standard(s) Supported
Math: 8.F.A.1, P.F.IF.A.8

Success Indicator
Learners will be successful if they:
• Calculate the Fibonacci sequence up to 8 digits
• Explain how the golden ratio, or phi, is calculated
• Can identify these patterns in items found in nature

Time Needed
30 - 45 minutes

Materials List
• Pencil and paper
• Graph paper
• Calculator
• Ruler
• A variety of objects (or pictures of objects) from nature: flowers, pinecones, pineapples, shells, fiddlehead ferns, etc.

Introduction to Content

In this lesson, students will calculate a Fibonacci sequence and identify examples of these numbers from nature. They will apply these numbers to draw a Fibonacci (or golden) spiral, and learn about the Golden Ratio.

Introduction to Methodology

There are three parts to this lesson. In part I, students will calculate the Fibonacci numbers and find examples of these in nature. Then there are two optional extensions. In part II, they will use these numbers to draw a Fibonacci spiral. In part III, they will calculate phi, or the golden ratio, and explore proportions in nature. Parts II and III are not dependent on each other, so you can do either, or both.

Author

Jennifer DeBruyn
Associate Professor, Department of Biosystems Engineering and Soil Science, University of Tennessee

Terms and Concepts Introduction

Fibonacci numbers – Named after an Italian mathematician (Leonardo Pisano Bigollo, born in 1170), this is a sequence of integers derived by summing the last two numbers to get the next number, or $F_n = F_{n-1} + F_{n-2}$

The Golden Ratio, or phi (ϕ) – An irrational number approximating 1.618 which describes the most aesthetically pleasing ratio.

Setting the Stage and Opening Questions

Show your students various flowers with three or five petals (can be pictures or real flowers).

Ask, **"How many petals do these flowers have? Do any of them have four petals? Have you ever seen a flower with four petals?"**

Say, **"Flowers with four petals are rare. In fact, four is an uncommon number in nature. Think about the four-leaf clover! We see three or five much more often. There are some patterns and numbers in nature that show up again and again. Today we're going to explore some of those numbers and see if we can find these patterns in nature."**

Tips for Engagement

Not everything you find outdoors will follow the Fibonacci numbers or golden ratios. Mutations and random chance or damage (e.g. a lost petal) will result in other numbers. Have several examples on hand of items that do follow the rules so the students can understand the patterns. Explain that these patterns are not universal, just very common.

READ: **"Blockhead" by Joseph D'Agnese** – a fictional interpretation of the life of Fibonacci.

Experience

Part I: Fibonacci sequence

Say, **"The first pattern we're going to explore is the Fibonacci sequence. This pattern of numbers was first recognized by an Italian mathematician in the 13th century. He noticed a sequence of numbers that came up again and again in nature that followed a pattern."**

1. Encourage the students to calculate the pattern on their own with pencil and paper.
   a. Fibonacci numbers start with the numbers 1 and 1
   b. To find the next number, add the previous two numbers: 1 + 1 = 2
   c. Now you have 1, 1, 2
   d. To find the next number, add the previous two numbers: 1 + 2 = 3
   e. Now you have 1, 1, 2, 3
   f. Continue on this pattern until you have at least 10 digits
   g. Final pattern should be: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…

2. Now that they know the numbers, have students identify items in nature that have Fibonacci numbers.
   a. You may start by showing them how our own body follows this numbers: Point out that we have 2 hands, 5 digits on each hand, 8 fingers, and 3 bones in each finger.
   b. Have them find their own items with Fibonacci numbers. Challenge them to find something for each number in the sequence up to 21.
      - If indoors, print off pictures of items in nature that have multiple structures like seeds or petals e.g. flower, leaves, pinecones, starfish etc. (e.g. Figure 1.)
      - If outdoors, have students conduct a scavenger hunt around the schoolyard to find items. Group these items by their number.

Experience

Part I: Fibonacci sequence (continued)

3. Point out another cool pattern with spirals. For items that have spiraling leaves or seeds, like pinecones, pineapples or sunflower heads, they will typically have two spirals going in opposite directions. If you count the number of items in each spiral, you will find that they have adjacent Fibonacci numbers. So for example, if your left hand spiral has 8 units, the right hand may have 13 (Figure 2). Try it out!

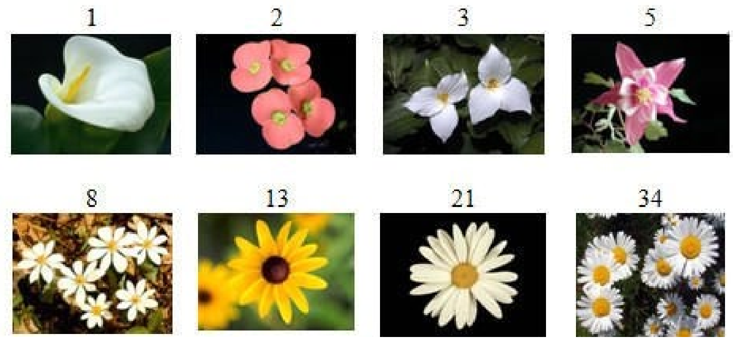

Figure 1. Flowers following Fibonnacci numbers.

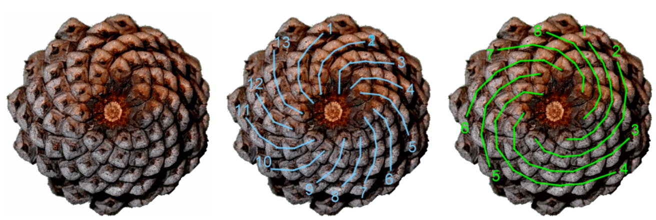

Figure 2. Opposite spirals on a pinecone. Image source: www.goldennumber.net/spirals/

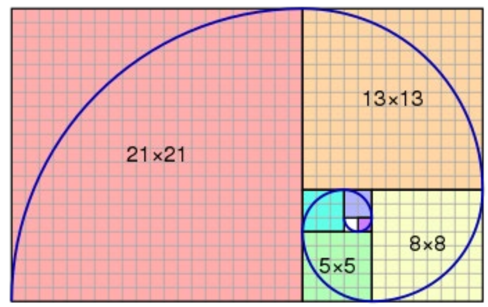

Figure 3. A Fibonacci spiral. Source: Wikipedia

Part II: Fibonacci spirals

Say, "**Fibonacci numbers don't just describe the number of things, but can also describe their arrangement. One pattern we see in nature is a spiral. Can you name anything that has a spiral shape?"** Possible answers include snail shells, sea shells, flowers, pineapples, and fiddleheads. Say, "**Believe it or not, these spirals are based on the Fibonacci numbers."**

1. Have your students draw a Fibonacci spiral (Figure 3) on a piece of graph paper. They can do this individually or get a giant piece of graph paper and do it as a class.
   a. With the graph paper in landscape orientation in front of you, count up from the bottom to the 6^{th} square and in from the right side to the 10^{th} square. Outline that square. This is the first 1.
   b. Move one square to the right, and outline this square. This is the second 1.
   c. Directly above those two squares, outline a 2 x 2 square.
   d. Directly to the left of those squares, outline a 3 x 3 square.
   e. Directly below, outline a 5 x 5 square. Continue moving around counter clockwise making an 8 x 8 square, then 13 x 13, then 21 x 21 until you run out of paper.

2. When all your boxes are outlined, start connecting the corners. On your first square, draw a line from upper left to lower right. On the second square, from lower left to upper right. On the 2 x 2 square, from lower right to upper left, etc.

3. *Optional extension:* Have students turn their spiral into art — draw a snail, a fern or whatever else they are inspired to create.

Experience

Part III: The Golden Ratio

Say, **"We're going to take the Fibonacci numbers a step further and calculate the Golden Ratio."**

1. Have your students take the Fibonacci numbers and divide each one by the previous number: 1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, 89/55, 144/89

2. This doesn't look like a pattern yet. But now, divide the numbers to get the decimal expansions. If they do this, they should get 1, 2, 1.5, 1.6666..., 1.6, 1.625, 1.615384615..., 1.619047619..., 1.617647059..., 1.618181818...

Say, **"Now see the pattern? The numbers get closer and closer to an irrational number: ($\sqrt{5}$ + 1)/2, which is approximately 1.6180339887, called phi (or ϕ). Phi is called the golden ratio. Other names include for this number include the golden section, golden mean, golden number, divine proportion, divine section and/or golden proportion. Like the underlying Fibonacci numbers, phi comes up in nature again and again."**

1. Have students find the golden ratio on their hands. You may want to show a picture of the hand bones so they understand what they are measuring.
   a. Measure the length of the bone at the end of your index figure (distal phalanges) from the tip of your finger to the middle of the first knuckle.
   b. Measure the second bone (intermediate phalanges) between first and second knuckle.
   c. Measure the third bone (proximal phalanges).
   d. Measure the hand bone (metacarpal) between the knuckle and the wrist

2. What numbers do you get? Calculate the ratio between the numbers.

Share

Ask, **"Did your hand and finger bones follow the golden ratio?"**

Process

Count the number of students in the class whose hand followed the ratio. Was it the majority of the class?

Life Skill(s)

- Demonstrate the ability to learn, reason, think creatively, make decisions, and problem solve.
- Acquire, communicate, organize, use and evaluate information.

Generalize

Ask, **"Why do you think these numbers appear over and over again in nature?"**
Scientists think that plants arrange in these patterns because it is the most efficient for packing (e.g. for seeds) or for maximizing sunlight exposure on leaves while minimizing shading of lower leaves.

Apply

Encourage your students to watch for "nature's secret code" around their own homes and schools. How many examples of Fibonacci sequence numbers or the golden ratio can they spot?

Supplemental Information
Educational Standards Met

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

P.F.IF.A.8 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.